COURSE SYLLABUS

1. COURSE DESCRIPTION

Degree:	**Administración y Dirección de Empresas** **(English teaching)**
Double Degree:	**Derecho y Administración y Dirección de Empresas** **(English teaching)**
Course:	**MATHEMATICS FOR BUSINESS II** **(Matemática Empresarial II - English teaching)**
Module:	**Quantitative Methods**
Department:	**Economics, Quantitative Methods and Economic History**
Term:	**Second term**
Total Credits:	**6**
Year:	**1st**
Type of Course:	**Obligatory**
Course Language:	**English**

Teaching model:	C1	
a. General/background:		50%
b. Theory-into-practice/developmental knowledge-building		50%
c. Guided Academic Activities:		

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3. ACADEMIC CONTEXT

3.1. Course Description and Objectives

Course desciption

Additional elements on Matrix Theory. Input-Output Analysis. Introduction to Optimization Theory (or Mathematical Programming) and its applications to Economics. Computer applications used for solving problems.

Objectives

- To make the student familiar with the daily mathematical vocabulary, and to make him used to a logic reasoning to carry out the resolutions of problems.
- To provide students with mathematical knowledge and techniques; these will be useful to complete their higher education and to carry out their professional life.
- To supply the student with the basic, indispensable tools from Optimization Theory (or Mathematical Programming) and Input-Output Analysis, so that he may be able to easily interpret and tackle mathematical models associated with the economic problems that can be found in other subjects and in the business world.
- To provide a deeper understanding of the computational software program *Mathematica*, and to facilitate the application of this computational tool for the resolution of problems posed within the course.

3.2. Contribution to the Training Plan

Mathematics for Business II is a 6-credit obligatory subject, belonging to the following plans: Business Administration and Management Degree, and Double Degree in Business Administration and Management, and Law. It is taught in the first year of both Degrees, and it depends on the Academic Area of Quantitative Methods in the Department of Economics, Quantitative Methods and Economic History (Departamento de Economía, Métodos Cuantitativos e Historia Económica).

Due to the contents of these Degrees, this course must be essentially practical, so that it can be applied to other subjects. The contents of the course have been selected considering the requirements of the others. We want to emphasize the utility of mathematical tools in other subjects, such as: *Microeconomía (Microeconomics), Estadística Empresarial I (Businees Statistics I), Estadística Empresarial II (Business Statistics II), Matemática Financiera (Financial Mathematics), Macroeconomía (Macroeconomics), Métodos Estadísticos y Econométricos en la Empresa (Statistical and Econometric Methods for Business), Modelos para la Programación y*

Planificación Empresarial (Business Programming and Planing Models) and *Técnicas Matemáticas de Decisión (Mathematical Techniques for Decision Making)*.

The importance of mathematical results explained in this course is rooted in their capacity to open different ways of solving problems from a range of different fields. The lecturers will show the straightest applications for each introduced topic, making the learning process more dynamic, and increasing the motivation of the student towards Mathematics.

In spite of the practical nature of this course, it cannot be reduced to a simple collection of methods to solve particular problems. Its formative nature must be taken into account, which will allow the student to develop skills in logic reasoning and in comprehension of formal language. It is necessary to make the student realize the importance of studying quantitative techniques because they are useful, but it is opportune to establish a minimum level of rigor which cannot be left aside under the pretext that Mathematics implies a basic knowledge for the economist and the businessperson.

3.3. Recommendations or Prerequisites

Although this is not a formal prerequisite, in order to understand the course, it is required to have basic knowledge from the Bachillerato and previous years. It would be also useful to have previously passed the course *Mathematics for Business I* (1st term, 1st year of Business Administration and Management Degree).

4. SKILLS

4.1. Degree Skills Developed during this Course

Instrumental:
- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
- Elementary computer skills and analytical abilities from different sources.
- Ability to solve problems and apply theory into practice.
- Decision making skills.

Personal:
- Ability to criticize and self-criticize.
- Ethical involvement at work.
- Ability to work under stressful conditions.

Systemic:
- Self-study and research skills.
- Ability to face up new situations.
- Awareness of quality and success.

Specific:
- Knowledge of basic mathematical techniques; application of the presented techniques in order to quantitatively analyze business situations.
- Ability to connect the concepts studied in this course with other subjects.

4.2. Module Skills Developed during this Course

Instrumental:
- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
- Elementary computer skills and analytical abilities from different sources.
- Ability to solve problems and apply theory into practice.
- Decision making skills.

Personal:
- Ability to criticize and self-criticize.

- Ethical involvement at work.
- Ability to work under stressful conditions.

Systemic:
- Self-study and research skills.
- Ability to face up new situations.
- Awareness of quality and success.

4.3. Course-specific Skills

- Knowledge of basic mathematical techniques in Input-Output Analysis through the Matrix Theory.
- To understand and put into practice the different optimization models.
- To be able to select and use the appropriate computing techniques in order to resolve such models.

5. COURSE CONTENT (COURSE TOPICS)

LEARNING MODULES
• Matrix Theory and Input-Output Analysis (Units 1 and 2).
• Convexity and Optimization Theory (Units 3 and 4).

COURSE TOPICS

UNIT 1: Input-Output Analysis
1.- Input-Output Production Model. Technical Matrix. Leontief Matrix.
2.- Input-Output Price Model.
3.- Productive Matrices: Characterization and Economic Interpretation.
4.- Autonomous Sets. Fundamental Products.
5.- Input-Output Analysis in the Andalusian Statistical System.

Description: The input-output model was introduced by Wassily Leontief, who was awarded the Nobel Prize in Economics in 1973. It uses a matrix representation of an economy to explain the relations between sectors or industries. It is a straightforward application of Matrix Theory, and allows analyzing the structure of the economy. So it is useful to determine the recommended total output for each sector to satisfy the final demand, but also to fix a price (value) for each product, or to know which sectors or industries can produce independently from the other sectors.

The more common mistakes that students make are caused by a lack of practice in matrix operations and solving systems of linear equations; both topics are covered in the course *Mathematics for Business I* (1st term, 1st year).

UNIT 2: Matrices in Business Modelling
1.- Eigenvalues and Eigenvectors of a Matrix. Characteristic Equation. Multiplicity.
2.- Diagonalizable Matrices. Similar Diagonal Matrix and Transformation Matrix.
3.- Quadratic Forms. Classification.
4.- Classification of Symmetric Matrices
5.- Computational Applications.

Description: In this unit, students will acquire the ability to deal with some elements from the Matrix Theory which will be useful in future units, when dealing with optimization of real-valued functions: eigenvalues, eigenvectors, quadratic forms and their classification.

The main problem the student finds in this unit is also to refresh the knowledge on matrix operations (mainly determinants) and systems of linear equations. It is absolutely essential to practice matrix operations as introduced in *Mathematics for Business I*.

UNIT 3: Differentiability and Convexity of Real-valued Functions
1.- Partial Derivatives. Marginal Rate of Technical Substitution.
2.- Higher-order Partial Derivatives. Hessian Matrix.
3.- Convex Sets. Convex and Concave Functions.
4.- Computational Applications.

Description: Convexity of sets and functions are very useful concepts in Optimization Theory, which will be covered in Unit 4. The study of these concepts is the main aim of this Unit 3. For this topic, it is crucial to master the computation of higher-order derivatives, which is covered in the first part of the unit.

The difficulties this unit involves are consequence of the lack of handling in the computation of derivatives of one variable functions, which obviously limit a proper computation of partial derivatives. The student will be aware of the importance of using these techniques skilfully, dedicating as much time as is needed before facing the challenge of more complex problems. It is also important to have properly understood the classification of quadratic forms, as covered in Unit 2.

UNIT 4: Optimization Theory
1.- Representation of the Problem. Concept of Optimum: Maxima and Minima, Strict and Non-strict Optima, Local (or Relative) and Global Optima. Local-Global Theorem. Weierstrass Theorem.
2.- Optimization of Real-valued Functions of One Real Variable (Single-variable Optimization).
3.- Optimization of Real-valued Functions of Several Variables (Multi-variable Optimization) without any Constraints.
4.- Optimization with Equality Constraints. Economic Interpretation of Lagrange Multipliers.
5.- Computational Applications.

Description: In this unit we deal with optimization problems. Problems with and without constraints will be solved. The solution here lies in choosing the best element from some set of available alternatives, and this is something useful even in daily life. When there is no constraint, the model can be used to assign economic resources to alternative objectives, while the existence of constraints may be due to the influence of budgetary restrictions. Our aim is to provide students with adequate techniques for each case.

The main difficulty for the student in this unit is the large amount of calculations needed in order to solve optimization problems. Derivatives computation, Hessian classification, and solving systems of equations (to find the critical points) can lead the student to awkward situations. Even the first representation of the problem can be an extra trouble when one is not able to express objective and constraints by means of real-valued functions and systems of equations, respectively.

6. METHODOLOGY AND RESOURCES

In order to improve the communication with students, on-site teaching will be supported by the virtual learning platform of the university.

As regards on-site sessions, they will be organized into the following two modules:

• General/background teaching (GT):

On-site sessions of 1.5 hours each (one per week) will be set throughout the whole term. These classes will be based on formal lessons taught by the lecturer and their main aim will be to introduce, in a schematic and general way, the main theoretical aspects of every unit. Sessions will be also devoted to solve problems on the blackboard by the lecturer.

• Practical/developmental knowledge-building teaching (PT):

These teachings will be also developed throughout on-site sessions, of 1.5 hours each, during the whole term. Lessons will be mainly practical and they will be devoted to solving problems by students. Some of these sessions will be hold in the computers room and directed by the lecturer. The final aim of these special sessions in the computers room (SSC) consists on providing students with the opportunity to learn how to use the symbolic computing software *Mathematica* so that they will apply it to solve similar problems to those set out during the other sessions. Specific notes about this program will be presented to students prior to SSC taking place, so that students could work on them beforehand.

Finally, personalized tutorials are optional for students. Lecturers will give students some guidance on their personal study, if they need so, to clarify specific doubts related to the subject contents, to correct wrong achieved concepts and habits, to make up some basic knowledge from students with a previous low-level and to provide additional bibliography. Every lecturer will provide some office hours for students, which will be communicated to them at the beginning of the course and they will be published on the virtual learning platform.

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7. ASSESSMENT

In general, for assessment purposes, all completed activities will be taken into account. They will be weighted differently in the final assessment depending on how important and difficult they are and how much effort has been put in by the students.

More precisely, the following specific tests will be carried out:

• Continuous evaluation (ongoing assessment):

Throughout the term, several tests and exercises will be carried out to follow student development in the acquisition of competences proposed in the Course. This ongoing assessment will take up 50% of the final mark, that is, 5 points out of 10. There will be various types of test:

- Theoretical knowledge will be tested at the end of each learning module through an on-line, multiple choice questionnaire. These exams will be taken through the virtual learning platform on the dates assigned at the time. The total value of these tests will be 1 point.

- At the end of each unit, and in the PT session which will be announced, the student will have to solve different exercises corresponding to the unit. These will be corrected and will have a total value of 2 points.

- Throughout the term, there will be three computer room sessions (SSC). The use of *Mathematica* software will be assessed within some specific sessions by solving some practical exercises using the computer, as well as practical exercises proposed by the lecturer and related to those given out to students before the computer room sessions. These exercises will have a total value of 2 points.

• Final exam:

This will be taken at the end of the term and will make up 50% of the final mark, that is, 5 points out of a total of 10. Theoretical knowledge will be evaluated through short questions or through multiple choice questions with a value of 1.5 points. Practical knowledge will be worth 3.5 points and will be tested by resolving problems.

To pass the course, the following minimal marks are needed:

- Computer sessions: 1 point out of 2.
- Final exam: 1.5 points out of 5.

Should the student not reach the minimum mark needed in the computer sessions throughout the term, he must pass a test (about these contents) on the final exam day.

If the minimal marks are reached, the final mark of this course will be the addition of the marks obtained in the written exam and in the ongoing assessment. A total of 5 points is needed to pass this course.

Students who have failed the course in the first summons (June) will have a second chance (July). In this second summons, the mark obtained in the ongoing assessment will also be considered. If the computer sessions had not been completed (with a final mark of at least 1 point out of 2) after the first summons, the student will also be tested on them (so, the final exam will include a computer test).

When taking exams (either the 'minimal knowledge' or the written exam), students will not be allowed to use reference or support materials.

To sit exams, students must be officially identified through their ID or another official form of identification.

Student mobility:

Those UPO students who are not able to attend seminars due to being abroad under official mobility programs (Socrates-Erasmus, Séneca, Atlanticus...) will have an additional exam, or work that will be clearly defined, in order to obtain the 50% of the grade corresponding to the continuous evaluation. Students in this situation must inform the responsible lecturers before the 30th of April. In case of not respecting this deathline, their academic coordinator should provide a validation document.

8. BIBLIOGRAPHY

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