

## COURSE SYLLABUS

### 1. COURSE DESCRIPTION

<b>Degree:</b>	<b>Administración y Dirección de Empresas (English teaching)</b>
<b>Double Degree:</b>	<b>Derecho y Administración y Dirección de Empresas (English teaching)</b>
<b>Course:</b>	<b>MATHEMATICS FOR BUSINESS I (Matemática Empresarial I - English teaching)</b>
<b>Module:</b>	<b>Basic Training in Business and Economic Sciences</b>
<b>Department:</b>	<b>Economics, Quantitative Methods and Economic History</b>
<b>Term:</b>	<b>First term</b>
<b>Total Credits:</b>	<b>6</b>
<b>Year:</b>	<b>1<sup>st</sup></b>
<b>Type of Course:</b>	<b>Basic</b>
<b>Course Language:</b>	<b>English</b>

<b>Teaching model:</b>	<b>C1</b>	
<b>a. General/background:</b>		<b>50%</b>
<b>b. Theory-into-practice/developmental knowledge-building</b>		<b>50%</b>
<b>c. Guided Academic Activities:</b>		

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### 3. ACADEMIC CONTEXT

#### 3.1. Course Description and Objectives

##### COURSE DESCRIPTION

Basic elements of Linear Algebra and Matrix Theory. Matrix operations. Basic elements of functions: continuity, differentiability and integration.

##### OBJECTIVES

- To provide students with mathematical knowledge and techniques; these will be useful to complete their higher education and to carry out their professional life.
- To supply the student with the basic, indispensable tools needed in order to easily interpret and tackle mathematical models associated with the economic problems that can be found in other subjects and in the business world.
- To give elemental tools from Linear Algebra, Matrix Theory, and Elements of Functions, to facilitate the comprehension of economic results.
- To make the student familiar with the daily mathematical vocabulary, and to make him used to a logic reasoning to carry out the resolutions of problems.
- To introduce the student to the use of the computational software program *Mathematica*, and to facilitate the application of this computational tool for the resolution of problems posed within the course.

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### 3.2. Contribution to the Training Plan

*Mathematics for Business I* is a 6-credit core subject, belonging to the following plans: Business Administration and Management Degree, and Double Degree in Business Administration and Management, and Law. It is taught in the first year of both Degrees, and it depends on the Academic Area of Quantitative Methods in the Department of Economics, Quantitative Methods and Economic History (Departamento de Economía, Métodos Cuantitativos e Historia Económica).

Due to the contents of these Degrees, this course must be essentially practical, so that it can be applied to other subjects. The contents of the course have been selected considering the requirements of the others. We want to emphasize the utility of mathematical tools in other subjects, such as: *Introducción a la Economía (Introduction to Economics)*, *Economía de la Empresa (Business Administration)*, *Microeconomía (Microeconomics)*, *Matemática Empresarial II (Mathematics for Business II)*, *Estadística Empresarial I (Business Statistics I)*, *Estadística Empresarial II (Business Statistics II)*, *Matemática Financiera (Financial Mathematics)*, *Macroeconomía (Macroeconomics)*, *Métodos Estadísticos y Econométricos en la Empresa (Statistical and Econometric Methods for Business)*, *Modelos para la Programación y Planificación Empresarial (Business Programming and Planning Models)* and *Técnicas Matemáticas de Decisión (Mathematical Techniques for Decision Making)*.

The importance of mathematical results explained in this course is rooted in their capacity to open different ways of solving problems from a range of different fields. The lecturers will show the straightest applications for each introduced topic, making the learning process more dynamic, and increasing the motivation of the student towards Mathematics.

In spite of the practical nature of this course, it cannot be reduced to a simple collection of methods to solve particular problems. Its formative nature must be taken into account, which will allow the student to develop skills in logic reasoning and in comprehension of formal language. It is necessary to make the student realize the importance of studying quantitative techniques because they are useful, but it is opportune to establish a minimum level of rigor which cannot be left aside under the pretext that Mathematics implies a basic knowledge for the economist and the businessperson.

### 3.3. Recommendations or Prerequisites

Although this is not a formal prerequisite, in order to understand the course, it is required to have basic knowledge from the Bachillerato and previous years. Especially for those students who do not have a *Bachillerato de Ciencias*, it would be appropriate a first stage where contents will be leveled.

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### 4. SKILLS

#### 4.1. Degree Skills Developed during this Course

##### Instrumental:

- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
- Elementary computer skills and analytical abilities from different sources.
- Ability to solve problems and apply theory into practice.

##### Personal:

- Ability to criticize and self-criticize.
- Ethical involvement at work.
- Ability to work under stressful conditions.

##### Systemic:

- Self-study and research skills.
- Ability to face up new situations.
- Awareness of quality and success.

##### Specific:

- Knowledge of basic mathematical techniques in Linear Algebra and Calculus; application of the presented techniques in order to resolve specific problems in business.
- Ability to connect the concepts studied in this course with other subjects.

#### 4.2. Module Skills Developed during this Course

##### Instrumental:

- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
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### Systemic:

- Self-study and research skills.
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### **4.3. Course-specific Skills**

- Knowledge of basic mathematical techniques in Linear Algebra and Calculus; application of the presented techniques in order to resolve specific problems in business.

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### 5. COURSE CONTENT (COURSE TOPICS)

#### LEARNING MODULES

- Linear Algebra and Matrix Theory (Unit 1)
- Differential Calculus and Integral Calculus, in one or several variables (Units 2 to 4)
- Sequences and Series (Unit 5)

#### COURSE TOPICS

##### ***UNIT 1: Basic Elements on Linear Algebra and Matrix Theory***

- 1.- *Representation of Economic Data through Real Matrices. Types of Matrices and Matrix Operations.*
- 2.- *Considering Variables of Several Dimensions. Vector Operations. Linear Dependence and Linear Independence.*
- 3.- *Linear Models of Several Equations. Solving and Interpreting Systems of Linear Equations.*
- 4.- *Computer Applications.*

Description: Many economic models can be formulated in terms of systems of linear equations, for example the static and dynamic models of equilibrium or the multisectorial models. All these formulations as well as Matrix Theory are used in the applied econometric models, simplifying them considerably, at times. From a historic point of view, Matrix Theory has been developed due to the need of studying methods to solve such systems.

The main purpose of this unit is to enable the student to solve any linear equation system that he can find. The fact that the student will be dealing with these systems later on in this subject and in others makes it absolutely essential to be able to handle them. This unit will therefore be treated in a practical way. In the setting out and solutions of the systems of equations presented here, matrices are extremely important, because they simplify the analysis of consistency and the determination of the number of solutions it has.

All concepts in this unit are neither new to the student nor extremely complex. Therefore those who work daily and follow the explanations well will feel at ease handling these tools.

##### ***UNIT 2: Real-valued Functions of One Real Variable***

- 1.- *Analysis of Basic Functions in the Context of Economics, Business and Management. Domain, Continuity and Graphical Representation.*
- 2.- *Absolute Variation and Relative Variation.*

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- 3.- *Computing Derivatives of Functions of One Real Variable. Higher Derivatives. Economic Interpretation.*
- 4.- *Computer Applications.*

Description: In the world of economy it is fundamental to establish relations between the different economic variables which allow us to easily analyze the influence from one to other. Such relations are usually expressed through functions and the changes produced in the variables are studied with the help of the concept of derivative. In this second unit special attention is paid to the real functions of one variable as the previous step to analyze the functions of several variables, which will be seen in the third unit. So, the main aim of this unit is to provide students with a deeper understanding of the computation of derivatives as well as other questions.

The main challenge the student will face in this unit is to master the computation of derivatives. Although it is an aspect which, in general, has been looked at in Bachillerato, some students have not practiced enough and need to devote some time to it. In order to compute derivatives of one variable functions, it is absolutely essential to go ahead with the partial derivatives which will be introduced in the next unit, and these, on the other hand, are essential to solve optimization problems, faced in the future.

### ***UNIT 3: Functions of Several Variables***

- 1.- *Real-valued Function. Functions in Economics. Real-valued Function of Several Variables. Graph of a Function. Some Type of Functions According to Their Economic Interpretation. Contour Lines (or Level Set).*
- 2.- *Continuity. Properties.*
- 3.- *Partial Derivatives. Gradient. Marginal Values. Elasticity. Interpretation.*
- 4.- *Homogeneous Functions.*
- 5.- *Computer Applications.*

Description: In the world of business and economics, it is common to express the relations between different economic variables through several variable functions. As common examples, we can point out: utility function, which measures the level of satisfaction which the consumption of an amount of goods gives to an individual; production function, which indicates the level of production obtained from the amount used; costs functions, which allow to calculate the cost associated to the process of production, knowing the amount of each of the productive factors which have been employed; etc.

In this unit, some of the concepts previously introduced are widened and the main purpose here is that the student acquires the ability to deal with functions of several variables and with the computation of partial derivatives, relating them to different economic aspects. We also explain homogeneous functions since they are fundamental in the Economic Theory. Economic Analysis supplies many examples which naturally hold homogeneous functions: productive processes with increasing or decreasing scale

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outputs, some demand functions, production functions, etc.

Taking into account the problems which this unit involves, the most significant one is the lack of handling in the computation of derivatives of one variable functions which obviously limit a proper computation of partial derivatives. The student will be aware of the importance of a good use of these techniques, dedicating to it as much time as needed.

### ***UNIT 4: Integration***

- 1.- *Primitive and Indefinite Integral. Computing Primitives.*
- 2.- *Definite Integral: Interpretation and Properties. Barrow's Rule.*
- 3.- *Improper Integrals.*
- 4.- *Double Integral. Integrals on General Regions.*
- 5.- *Computer Applications.*

Description: Unit 4 deals with simple and multiple integrations. The student must become competent in handling of these integration techniques in order to apply them later to several variables functions or to other subjects such as Microeconomics and Statistics. In these subjects, these tools are very useful to put into practice some concepts: the surplus of the consumer, and the determination of probabilities for bidimensional random variables from the probability density function.

To make a distinction from the computation of derivatives, which is mainly mechanical, the computation of primitives is much more complex. The most common methods, previously described, can help the student to tackle the computation, but only dedication and time will enable the student to face them easily.

### ***UNIT 5: Sequences and Series***

- 1.- *Numerical Sequence. Limits of Sequences. Convergent and Divergent Sequences. Computing of Limits.*
- 2.- *Numerical Series: Series with All their Terms Positive.*
- 3.- *Addition of Series: Arithmetic Series and Geometric Series.*
- 4.- *Functional Sequences. Convergence.*
- 5.- *Computer Applications.*

Description: The first part of the unit is devoted to the study of numerical sequences, especially the methods to compute limits of those which are convergent (with a finite limit), as an introduction to the study of numerical series. Arithmetic and geometric series are especially interesting due to their utility in the computation of actual and real values of financial rents, which will be studied further ahead in the subject of Financial Mathematics.

The concepts introduced in this unit are perhaps the most unfamiliar to students. These concepts can seem too abstract and it creates a more difficult challenge for their assimilation. Anyway, we try to deal with them easily and to complete the unit in a practical way, in order to reduce their complexity.

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### 6. METHODOLOGY AND RESOURCES

On-line and on-site teaching will be combined in the development of the subject.

The theoretical part, in which the basic aspects of the subject will be shown, should be taken by the students mainly on-line, using the WebCT platform, according to the schedule specified by the lecturers. On-site classes are mostly intended to solve problems related to Business Sciences, highlight their right understanding and apply the right mathematical theorems and make the subject as practical as possible.

As regards on-site teaching, it will be organized into the following two modules:

- General/background teaching (GT):

On-site sessions of 1.5 hours each (one per week) will be set throughout the whole term. These classes will be based on formal lessons taught by the lecturer and their main aim will be to introduce, in a schematic and general way, the main theoretical aspects of every unit, so that students can deal with their on-line development by themselves later on. Sessions will be also devoted to solve problems on the blackboard by the lecturer. To do this, skills developed in theoretical lessons and on-line will be applied.

- Practical/developmental knowledge-building teaching (PT):

These teachings will be also developed throughout on-site sessions, of 1.5 hours each, during the whole term. Lessons will be mainly practical and they will be devoted to solving problems by students. Some of these sessions will be hold in the computers room and directed by the lecturer. The final aim of these special sessions in the computers room (SSC) consists on providing students with the opportunity to learn how to use the symbolic computing software *Mathematica* so that they will apply it to solve similar problems to those set out during the other sessions. Specific notes about this program will be presented to students prior to SSC taking place, so that students could work on them beforehand.

Finally, personalized tutorials are optional for students. Lecturers will give students some guidance on their personal study, if they need so, to clarify specific doubts related to the subject contents, to correct wrong achieved concepts and habits, to make up some basic knowledge from students with a previous low-level and to provide additional bibliography. Every lecturer will provide some office hours for students, which will be communicated to them at the beginning of the course and they will be published on the WebCT.

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### 7. ASSESSMENT

In general, for assessment purposes, all completed activities will be taken into account. They will be weighted differently in the final assessment depending on how important and difficult they are and how much effort has been put in by the students.

More precisely, the following specific tests will be carried out:

- Final exam:

This will be taken at the end of the term and will make up 50% of the final mark, that is, 5 points out of a total of 10. Theoretical knowledge will be evaluated through short questions or through multiple choice questions with a value of 1.5 points. Practical knowledge will be worth 3.5 points and will be tested by resolving problems.

- Continuous evaluation (ongoing assessment):

Throughout the term, several tests and exercises will be carried out to follow student development in the acquisition of competences proposed in the Course. This ongoing assessment will take up 50% of the final mark, that is, 5 points out of 10. There will be various types of test:

- Theoretical knowledge will be tested at the end of each unit through an on-line, multiple choice questionnaire. These exams will be taken through the WebCT platform on the dates assigned at the time. The total value of these tests will be 1 point.
- At the end of each unit, and in the PT session which will be announced, the student will have to solve different exercises corresponding to the unit. These will be corrected and will have a total value of 2 points.
- Throughout the term, there will be three computer room sessions (SSC). The use of *Mathematica* software will be assessed within these sessions by solving some practical exercises using the computer, as well as practical exercises proposed by the lecturer and related to those given out to students before the computer room sessions. These exercises will have a total value of 2 points.

To pass the course, the following minimal marks are needed:

- Final exam: 1.5 points out of 5.
- Computer sessions: 1 point out of 2.

Should the student not reach the minimum mark needed in the computer sessions throughout the term, he must pass a test (about these contents) on the final exam day.

If the minimal marks are reached, the final mark of this course will be the addition of the marks obtained in the written exam and in the ongoing assessment. A total of 5 points is needed to pass this course.



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Students who have failed the course in the first term (January-February) will have a second chance (June-July). In this second summons, the mark obtained in the ongoing assessment will also be considered. If the computer sessions had not been completed (with a final mark of at least 1 point out of 2), the student will also be tested on them (so, the final exam will include a computer test).

When taking exams (either the 'minimal knowledge' or the written exam), students will not be allowed to use reference or support materials.

To sit exams, students must be officially identified through their ID or another official form of identification.

### Student mobility:

Those UPO students who are not able to attend seminars due to being abroad under official mobility programs (Socrates-Erasmus, Séneca, Atlanticus...) will have an additional exam, or work that will be clearly defined, in order to obtain the 50% of the grade corresponding to the continuous evaluation. Students in this situation must inform the responsible lecturers before the 30<sup>th</sup> of November. In case of not respecting this deadline, their academic coordinator should provide a validation document.

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### 8. BIBLIOGRAPHY

#### GENERAL READING

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LARSON, R.E.; HOSTETLER, R.P.; EDWARDS, B.H. Calculus with Analytic Geometry. McGraw-Hill. London.

#### FURTHER READING

##### PREREQUISITES:

LARSON, R; HOSTETLER, R.P. Algebra for College Students. Houghton Mifflin Company, 2005.

WEISSTEIN, E.W. CRC Concise Encyclopedia of Mathematics. 2nd edition. Chapman & Hall/CRC, 2002.

##### LINEAR ALGEBRA:

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### DIFFERENTIAL CALCULUS & INTEGRAL CALCULUS:

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BIRCHENHALL, C; GROUTH, P. Mathematics for Modern Economics. Phillips Allan, 1984.

GOLDSTEIN, L; LAY, D; SCHNEIDER, D. Calculus and its Applications. Prentice-Hall International Editions, 1993.

KHOURY, J; PARSONS, T. Mathematical Methods in Finance and Economics. North Holland, 1981.

KLEIN, M.W. Mathematical Methods for Economics. Addison Westley Reading Mass, 1997.

NICHOLSON, R.H. Mathematics for Business and Economics. McGraw Hill, 1986.

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### SEQUENCES & SERIES:

BLACK, A; BRADLEY, W. Essential Mathematics for Economists. Wiley and Sons, 1975.

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WOLFRAM, S.: The Mathematica Book. Ed. Cambridge University Press. Campiagn, 2003.

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